Reliability-Based Design Optimization using Kriging and Support Vector Machines

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ABSTRACT: This article presents a novel approach for Reliability-Based Design Optimization (RBDO) using Kriging and Support Vector Machines (SVMs). The proposed algorithm is based on a sequential two level scheme. The first stage consists of solving an approximated probabilistic optimization problem. The objective function and the failure domains are approximated by Kriging and SVMs respectively. The probability of failure and its sensitivity are estimated using subset simulation. The availability of the sensitivity allows one to solve the subproblem using a gradient-based method. The second level deals with the local refinement of the failure domains approximations around the first stage subproblem solution. In the second stage, a key contribution of this work is the use of a novel probabilistic "max-min" sample that refines the failure boundary based on the random variable distributions as well as the locations of the samples. The proposed scheme is applied to three test cases including an analytical example featuring a failure domain defined by 100 dummy failure modes and a crash-worthiness analysis featuring 11 dimensions and 10 failure domains.

1 INTRODUCTION

Reliability-based design optimization (RBDO) differs from deterministic optimization in the sense that the objective and/or constraints are probabilistic. As a consequence, the traditional hurdle due to computationally intensive black box models might be greatly amplified because of the need to evaluate a probability of failure. This difficulty is further increased when there are a large number of nonlinear limit states.

In an effort to make RBDO suitable for engineering applications, several approaches have been developed to tackle the reliability aspect. The most popular ones are Reliability Index Approach (RIA), Performance Measure Approach (PMA) formulations (Tu, Choi, & Park 1999), and Sequential Optimization and Reliability Assessment (SORA Du & Chen 2004). These methods are based on first order reliability method (FORM) which is based on the hypothesis that the limit state is linear or weakly non-linear. Although such assumptions might hold reasonably well for some engineering applications, there are many cases (e.g., system reliability or highly non-linear limit states) where this will not be true and might lead to unsafe designs.

In order to deal with system reliability or highly non-linear limit states, sampling-based techniques such as Monte-Carlo simulations are more appropriate. However the computational cost involved in such process make it impractical.

Examples of solution strategies that have been widely investigated to tackle this limitation consists of using surrogates of the limit state such as Kriging (Bichon, Mahadevan, & Eldred 2009) or approximations of the failure domain using a support vector machine (SVM) (Basudhar & Missoum 2010). However the variability of the sampling-based probability estimates and the associated non-differentiability of the probabilistic constraints make the use of gradient-based optimization techniques impractical. Fortunately, approximations of the sensitivities as a by-product of most sampling-based techniques have been derived (Song, Lu, & Qiao 2009, Lebrun & Dutfoy 2009, Dubourg, Sudret, & Bourinet 2011). In most cases, the surrogates or classifiers are refined globally and used in a nested RBDO loop using the aforementioned sensitivities (Dubourg, Sudret, & Bourinet 2011). Another strategy uses the sensitivities in a decoupled approach (Zou & Mahadevan 2006).

The proposed work is based on the fact that the approximation of the failure domain needs to be accurate only in the surrounding of the actual optimum. In this work, the optimum is searched through a sequence of optimization subproblems which minimize the approximation of the objective function subjected to probabilistic constraints. While the objective function is approximated using Kriging, an SVM classifier is used to approximate the boundaries of the failure domain. Following the solution of a subproblem, the SVM approximation is refined using a dedicated adaptive sampling scheme around the current optimum.

The main features of the proposed algorithm are:

- Once the SVM is constructed, the probability of failure is evaluated using Subset Simulations (Au & Beck 2001);
- The solution of the subproblem is made possible because of the availability of the sensitivity of the probability of failure (Song, Lu, & Qiao 2009) thus allowing the use of gradient-based optimization techniques;
- The use of a classifier to approximate the failure domain allows one to treat problems with discontinuities. In addition, only one SVM might be needed per failure domain even if it is defined through a large number of limit states;
- The refinement of the failure domain approximation is made through a new scheme that simultaneously encompasses the distributions of the variables as well as the location of the samples.

The paper is structured as follows: Section 2.1 provides an overview of the main steps of the approach. Section 2.4 introduces the optimization subproblem. Section 2.5 explains the local refinement scheme to refine the boundary of the failure domain. Section 2.7 lists the various metrics considered to monitor the convergence of the algorithm. In the results section 3, the approach is applied to three example: an analytical problem featuring a failure domain made of 100 dummy failure modes, a 6 dimensional study of a short column and a 11 dimensional crash-worthiness analysis featuring 10 domain of failures.

2 PROPOSED APPROACH

This article introduces a method to solve the following RBDO problem:

$$\begin{aligned} \min_{\boldsymbol{\theta}} \quad & F(\boldsymbol{\theta}) \\ \text{s.t.} \quad & \mathbb{P}[\mathbf{X} \in \Omega_{F_i}] \leq \mathbb{P}_{T_i} \quad i = 1, \ldots, n_c \\ & \mathbf{L} \leq \boldsymbol{\theta} \leq \mathbf{U} \end{aligned} \tag{1}$$

where $\boldsymbol{\theta}$ are the hyper-parameters (e.g., means) of the distributions of the d random variables $\mathbf{X}$, n_c is the number of probabilistic constraints, and $\mathbf{L}$ and $\mathbf{U}$ are the lower and upper bound of $\boldsymbol{\theta}$ respectively. Ω_{F_i} is a failure domain for a series or parallel system (defined by a set of limit states $g_{i,j}$) and $\mathbb{P}_{T_i}$ is the corresponding target probability. The algorithm focuses on problems with a computationally expensive objective function (F) and nonlinear limit states (potentially discontinuous and in large number).

2.1 *Summary of the approach*

The proposed RBDO algorithm is based on a succession of subproblems constructed from approximations of the objective function and failure domain. The key aspect of the methodology is the use of an adaptive sampling scheme to refine the boundary of the failure domains involved in the estimation of the probabilities of failure defined in Eq. 1. The chart on Figure 1 provides a summary of the approach. The main features of the algorithm are developed further in the subsequent sections. The detailed algorithm is summarized in Algorithm 1.

2.2 *Choice of surrogates*

In the case where the objective function is computationally expensive, the objective function F is

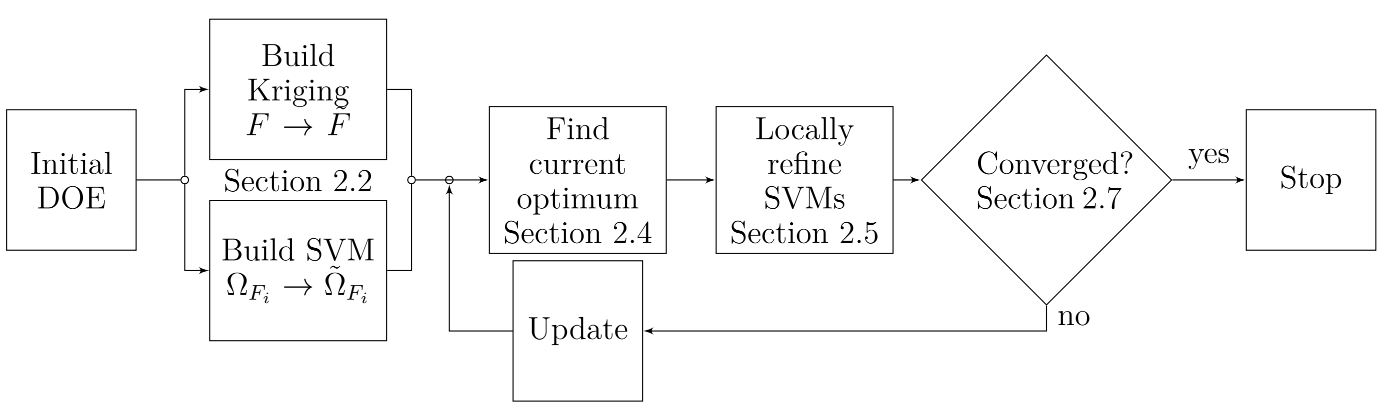


Figure 1: Overview of the proposed RBDO algorithm.

replaced by a Kriging approximation ($\tilde{F}$) (Sacks, Welch, Mitchell, & Wynn 1989, Jones 2001, Forrester & Keane 2009, Basudhar, Dribusch, Lacaze, & Missoum 2012).

The probabilistic constraints are treated by approximating the boundary of the failure domains. This is done using a support vector machine (SVM) classifier (Gunn 1998, Vapnik 2000, Schölkopf & Smola 2002, Christianini & Taylor 2000). The choice of SVM stems from its ability to handle discontinuous response and the possibility of using one single SVM to approximate the boundaries of a failure domain Ω_{F_i} defined by large number of limit states ($g_{i,j}$) for series system.

Note that the approach is not dependent on Kriging nor SVM. Any surrogate or classifier could be used within the proposed scheme.

2.3 *Probability of failure*

In the case of highly nonlinear limit states, moment based reliability techniques such as FORM or SORM are not suitable. Sampling techniques are typically favored. Among such techniques, Subset Simulations (Au & Beck 2001) have shown good performances, and are used in this work. They are particularly useful for small target probabilities and also enable a reduction of the variance of the probability estimate. Another possible choice would be Multi-modal Adaptive Importance Sampling (Zou & Mahadevan 2006).

2.4 *Sub-problem definition*

The probabilistic optimum at iteration k is found by solving the following optimization problem:

$$\boldsymbol{\theta}_{new}^{(k)} = \arg\min_{\boldsymbol{\theta}} \quad \tilde{F}^{(k)}(\boldsymbol{\theta}) \qquad (2)$$

$$\text{s.t.} \quad \mathbb{P}\left[\mathbf{X} \in \tilde{\Omega}_{F_i}^{(k)}\right] \leq \mathbb{P}_{T_i}$$

where $\tilde{\Omega}_{F_i}$ is the SVM-based approximation of the failure domain Ω_{F_i}. Due to the variance inherent to sampling techniques, the probabilistic constraints are noisy and non-differentiable numerically (Missoum, Ramu, & Haftka 2007). However, it can be shown that the sensitivities:

$$\frac{\mathrm{d}P_f}{\mathrm{d}\theta_i} = \int_{\mathbf{x}\in\Omega_f} \frac{\mathrm{d}\mathbf{f}_{\mathbf{x}}(\mathbf{x}|\boldsymbol{\theta})}{\mathrm{d}\theta_i}\mathrm{d}\mathbf{x}$$

can be efficiently approximated as a by product of sampling techniques (Song, Lu, & Qiao 2009, Lebrun & Dutfoy 2009, Dubourg, Sudret, & Bourinet 2011). For example, if $X \sim N(\theta, 1)$:

$$\frac{\mathrm{d}P_f}{\mathrm{d}\theta} = \mathbb{E}\left[X | X \in \Omega_f\right]$$

Based on the availability of the derivatives, sub-problem defined in Eq. 3 can be solved using a gradient based optimization technique such as SQP.

2.5 *Refinement of the SVM*

The refinement of the SVMs is ensured through the use of a "probabilistic max-min" sample defined as:

$$\boldsymbol{\theta}_{MM_i}^{(k)} = \arg\max_{\mathbf{x}} \quad \boldsymbol{\phi}(\mathbf{x}|\boldsymbol{\theta}_{new}^{(k)})^{\frac{1}{d}} d_{nearest}(\mathbf{x}) \qquad (3)$$

$$\text{s.t.} \quad \mathbf{x} \in \partial\tilde{\Omega}_{F_i}^{(k)}$$

where:

$$d_{nearest}(\mathbf{x}) = \min_i(||\mathbf{x} - \mathbf{x}^{(i)}||)$$

and $\mathbf{x}^{(i)}$ are the samples already found, $\partial\tilde{\Omega}_{F_i}^{(k)}$ the approximated boundary of the i^{th} failure domain, $\boldsymbol{\phi}(\mathbf{x}|\boldsymbol{\theta}_{new}^{(k)})$ is the d-dimensional standard normal joint distribution with hyper-parameters $\boldsymbol{\theta}_{new}^{(k)}$.

Note that the proposed formulation is an important variation on the previous work by Basudhar & Missoum who introduced a "max-min" search for the refinement of SVMs which only considered spatial distributions of the samples. In order to compare both schemes, consider the following problem: within a hypercube of side $2a$ centered on the origin, sequentially fill the space using samples as defined by:

$$\mathbf{x} = \arg\max_{\mathbf{x}} \quad d_{nearest}(\mathbf{x}) \qquad (4)$$

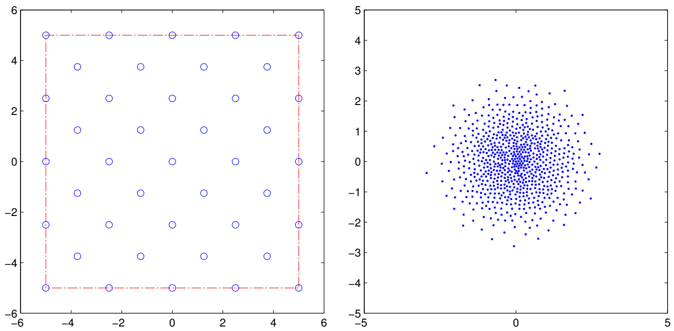
$$\text{s.t.} \quad a \leq x_i \leq a \qquad \forall i$$

Figure 2(a) shows the pattern of such a max-min filler in 2 dimensions. The authors claim that regardless of the number of points and the dimension, the marginal distribution of the samples along any axis is uniform between $-a$ and a.

Consider now this other setup. Sequentially fill the space using samples as defined by:

$$\mathbf{x} = \arg\max_{\mathbf{x}} \quad \boldsymbol{\phi}(\mathbf{x})^{\frac{1}{d}} d_{nearest}(\mathbf{x}) \qquad (5)$$

where $\boldsymbol{\phi}$ is d-dimensional standard normal joint probability density function. Figure 2(b) shows the pattern of such a probabilistic max-min filler in 2 dimensions. The authors claim that regardless to the number of points and the dimension, the samples follows the joint distribution $\boldsymbol{\phi}$. At that point, the authors have not been able to derive a rigorous proof of that characteristic. However, this was shown empirically up to 30 dimensions.



(a) Pattern of a max-min filler within a cube of side 10.

(b) Pattern of a probabilistic max-min filler.

Figure 2: Pattern in 2 dimension of two max-min filler.

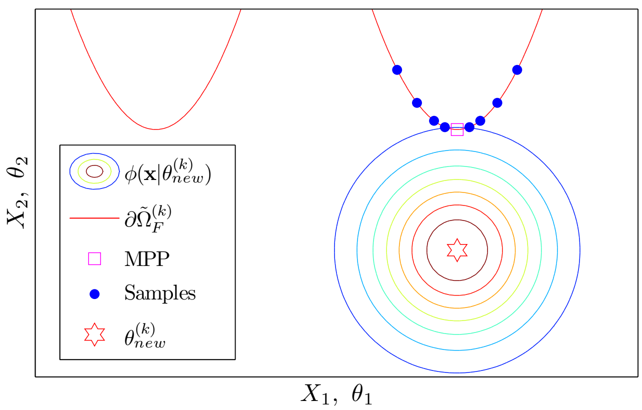


Figure 3: Probabilistic placement of point along the boundary of a failure domain according to the joint defined by $\boldsymbol{\theta}^{(k)}_{new}$.

2.6 *Properties of the probabilistic max-min for reliability assessment*

Consider the probabilistic sample as defined by Eq. 3. Figure 3 show the pattern of points that would be obtained for a joint PDF with hyper-parameters $\boldsymbol{\theta}^{(k)}_{new}$. It is noteworthy that the probabilistic samples enable the refinement of the failure domain while following the joint distribution $\boldsymbol{\phi}(\mathbf{x}|\boldsymbol{\theta}^{(k)}_{new})$. This refinement is made possible by simultaneously forcing the samples to be as far away as possible from each other.

Note that this approach is not restricted to normal distributions as isoprobabilist transformations can be used. In the case of correlated variables, Nataf transformation (Nataf 1962) or copulas (Nelsen 2006) can be used.

2.7 *Convergence metrics*

The use of a surrogate for the objective function and SVMs make the definition of a unique convergence criterion rather tedious. In this work, the following metrics are considered:

The overall convergence of the algorithm can be monitored through soft and hard convergence criteria:

$$\text{Soft} \quad : \quad \rho_S = \frac{\left\|\boldsymbol{\theta}^{(k)}_{new} - \boldsymbol{\theta}^{(k-1)}_{new}\right\|}{\left\|\boldsymbol{\theta}^{(k-1)}_{new}\right\|} \leq \varepsilon_s \tag{6}$$

$$\text{Hard} \quad : \quad \rho_H = \left|\frac{F(\boldsymbol{\theta}^{(k)}_{new}) - F(\boldsymbol{\theta}^{(k-1)}_{new})}{F(\boldsymbol{\theta}^{(k-1)}_{new})}\right| \leq \varepsilon_s \tag{7}$$

The convergence of the Kriging model is studied using a trust-region metric which is the ratio of the predicted improvement over the actual improvement:

$$\rho_F = \frac{\tilde{F}^{(k)}(\boldsymbol{\theta}^{(k)}_{new}) - \tilde{F}^{(k)}(\boldsymbol{\theta}^{(k-1)}_{new})}{F(\boldsymbol{\theta}^{(k)}_{new}) - F(\boldsymbol{\theta}^{(k-1)}_{new})} \tag{8}$$

When ρ_F is positive, the surrogate exhibits the proper trend (predicted a reduction and observed a reduction). The closer from 1 this ratio is, the better the surrogate is.

The convergence of the SVMs is observed using the following ratio:

$$\rho_{\Omega_{F_i}} = \left|\frac{\beta_{i,k+1} - \beta_{i,k}}{\beta_i}\right| \tag{9}$$

where:

$$\beta_{i,k} = -\Phi^{-1}\left(\mathbb{P}\left[\mathbf{X} \in \tilde{\Omega}^{(k)}_{F_i}\right]\right) \ , \ \beta_i = -\Phi^{-1}\left(\mathbb{P}_{T_i}\right)$$

The advantage of this quantity stems from its ability to quantify the relative change in probability due to the update scheme. If this ratio tend to zero and ρ_S is small, then the refinement scheme do not improve the SVM anymore. Note that this does not guarantee that the SVM converged to the actual one but only that it is converged for the proposed refinement scheme.

3 RESULTS

3.1 *Multiple failure modes: Analytical example*

This first example is a simple analytical problem involving 100 dummy constraints. It is used to show the possible advantage of using a single SVM to represent a failure domain defined through a large number of limit states. The RBDO problem is defined as:

$$\min_{\theta_1,\ \theta_2} \quad \left(\frac{\theta_1}{15} + \frac{1}{2}\right)^2 + \sin\left[4\left(\frac{\theta_2}{15} - 1\right)^2\right] \tag{10}$$

$$\text{s.t.} \quad \mathbb{P}[\mathbf{X} \in \Omega_{F_1}] \leq 10^{-3}$$

$$0 \leq \theta_1 \leq 40$$

$$-5 \leq \theta_2 \leq 35$$

Algorithm 1 RBDO algorithm

Compute an initial DOE of size m:

$$Y^{(d)} = F(\boldsymbol{\theta}^{(d)}),\ G_i^{(d)} = g_i(\boldsymbol{\theta}^{(d)}) \quad d = [1, m]$$

Build initial Kriging $\tilde{F}^{(m)}$ and SVMs $\tilde{\Omega}_{F_i}^{(m)}$;
k=m+1;
while Not Converged (Section 2.7) **do**
  Find the current probabilistic optimum (Section 2.4): $\boldsymbol{\theta}_{new}^{(k)}$ (Eq. 3);
  Refine $\tilde{\Omega}_{F_i}$, $i = [1, n_c]$ (Section 2.5) by adding a probabilistic max-min: $\boldsymbol{\theta}_{MM_i}^{(k)}$ (Eq. 3);
  Update:
    1. $\tilde{F}^{(k)}$ into $\tilde{F}^{(k+1)}$ using $\boldsymbol{\theta}_{new}^{(k)}$;
    2. $\tilde{\Omega}_{F_i}^{(k)}$ into $\tilde{\Omega}_{F_i}^{(k+1)}$ using $\boldsymbol{\theta}_{MM}^{(k)}$;
  k=k+1;
end while

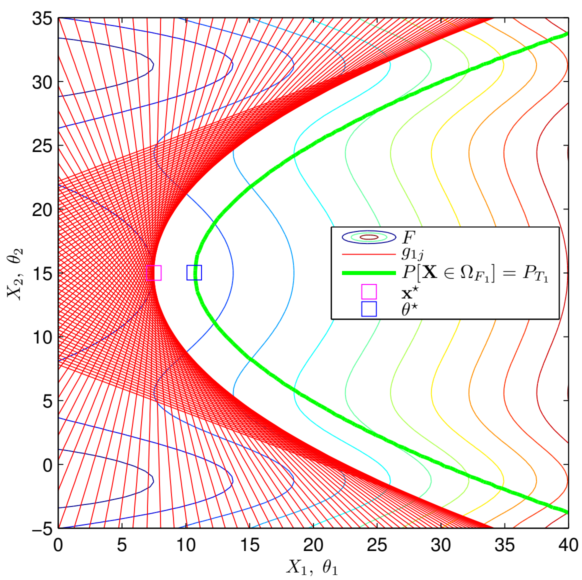


Figure 4: Graphical representation an analytical example featuring 100 dummy constraints.

where Ω_{F_1} is bounded by 100 dummy lines that are tangent to the parabola defined as:

$$-\left(\frac{x_2}{15} - 1\right)^2 + \frac{x_1}{15} - 0.5 = 0$$

and $X_i \sim N(\theta_i, 1)$. Figure 4 depicts the problem graphically. The initial design of experiments is a Central Voronoi Tessellation (CVT) DOE of 15 points. Each iteration adds one sample to refine the approximation of Ω_{F_1} and one sample for F. The convergence is shown over 20 iterations in Figure 5 and the results are summarized in Table 1.

In order to assess the reduction in the number of function evaluations using a single SVM, Figure 6 depicts the number of limit states that need to be computed for each max-min sample. The 15 first samples are from the initial DOE. In the case of a safe sample the total number of limit states (100)

Table 1: Results for the 100 constraints analytical problem at iteration 17

	Optimum	Actual*	Error (%)
θ_1	10.6623	10.6655	0.03
θ_2	14.8952	15.0285	0.89
F	1.4663	1.4666	0.03
$P_f^{\star\star}$	1.06×10^{-3}	10^{-3}	6
$\beta^{\star\star\star}$	3.07	3.09	0.65

* Obtained through a brute nested RBDO technique
** $P_f = \mathbb{P}[\mathbf{X} \in \Omega_{F_1}]$, *** $\beta = -\Phi\left(\mathbb{P}[\mathbf{X} \in \Omega_{F_1}]\right)$

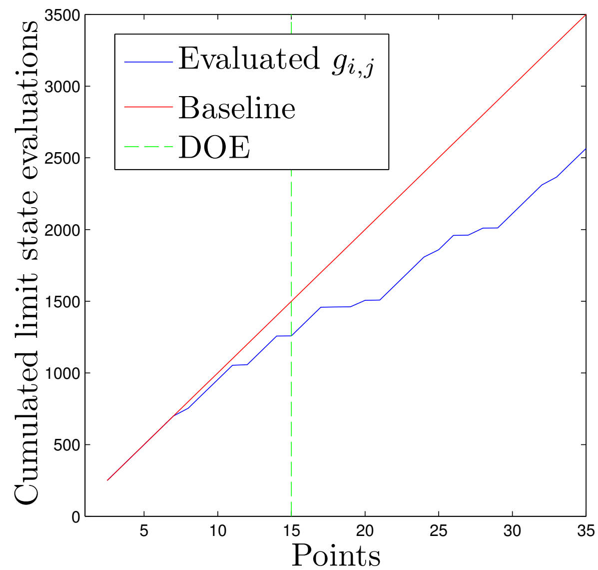


Figure 6: Amount of failure modes $g_{i,j}$ evaluated per points.

must be evaluated. At iteration 17, beyond which no further improvement is observed, 2311 limit state function calls have been performed, whereas 3200 would have been required if all failure modes were to be accounted for independently. The gain is of 28%. This improvement was obtained with a simple reordering of the limit states based on their criticality.

3.2 *6 dimensions: Short column*

This problem consists in the minimization of the weight of a short column with rectangular cross section of height h and width b (Aoues & Chateauneuf 2010). It is subjected to an axial load F and two bending moments about the two axes of inertia of the cross section. The design should satisfy stress requirements such that:

$$1 - \frac{4M_1}{bh^2 f_y} - \frac{4M_2}{b^2 h f_y} - \left(\frac{F}{bhf_y}\right)^2 \leq 0$$

which defines Ω_{F_1} based on the yield stress f_y. The random variables are listed in Table 2. There are a total of six random variables, two of which have their mean values θ_1 and θ_2 used as optimization variables.

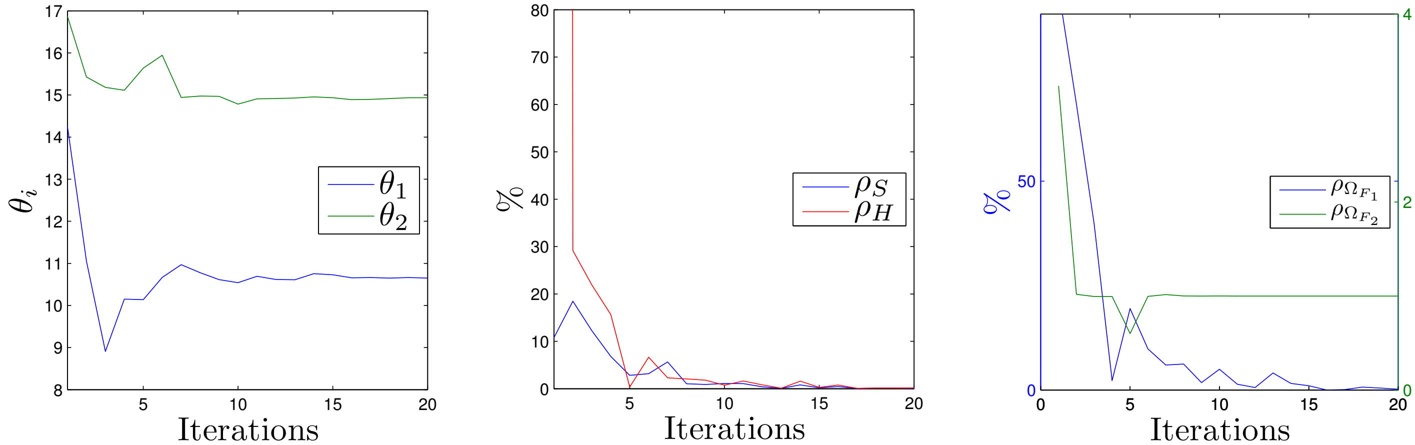


Figure 5: Convergence of the proposed algorithm for the 100 constraints analytical example (left to right: design variables, overall convergence, surrogates convergence).

The problem is defined as:

$$\min_{\theta_1,\ \theta_2} \quad \theta_1\theta_2 \tag{11}$$

$$\text{s.t.} \quad \mathbb{P}[\mathbf{X} \in \Omega_{F_1}] \leq \Phi(-3)$$

$$0 \leq \theta_1, \theta_2 \leq 1 \qquad \frac{1}{2} \leq \frac{\theta_1}{\theta_2} \leq 2$$

An initial CVT DOE of 60 points is used. The convergence of the algorithm over 70 iterations is shown on Figure 7 and the results are summarized in Table 3.

Table 2: Random variables for the short column problem

Variable	Distribution	Mean	C.o.V. (%)
b (m)	Normal	θ_1	10
h (m)	Normal	θ_2	10
M_1 (N.m)	Normal	250×10^3	30
M_2 (N.m)	Normal	125×10^3	30
F (N)	Normal	2.5×10^6	20
f_y (Pa)	Normal	4×10^7	10

Table 3: Results for the short column problem at iteration 60

	Optimum	Actual*	Error (%)
θ_1	0.3462	0.3586	3.44
θ_2	0.6925	0.6710	3.2
F	0.2398	0.2406	0.08
$P_f^{\star\star}$	1.07×10^{-3}	1.35×10^{-3}	20.7
$\beta^{\star\star\star}$	3.06	3	2

* Obtained through a brute nested RBDO technique
** $P_f = \mathbb{P}[\mathbf{X} \in \Omega_{F_1}]$, *** $\beta = -\Phi\left(\mathbb{P}[\mathbf{X} \in \Omega_{F_1}]\right)$

3.3 *11 dimensional problems with multiple failure domains: Crash-worthiness*

This problem deals with the crash-worthiness analysis of a car subjected to a side impact. This work uses the problem definition from Youn, Choi, Yang, & Gu 2004, Thoomu 2010 where responses surfaces have been obtained to approximate the constraints and objective function. In this work, these responses surfaces are used as black boxes.

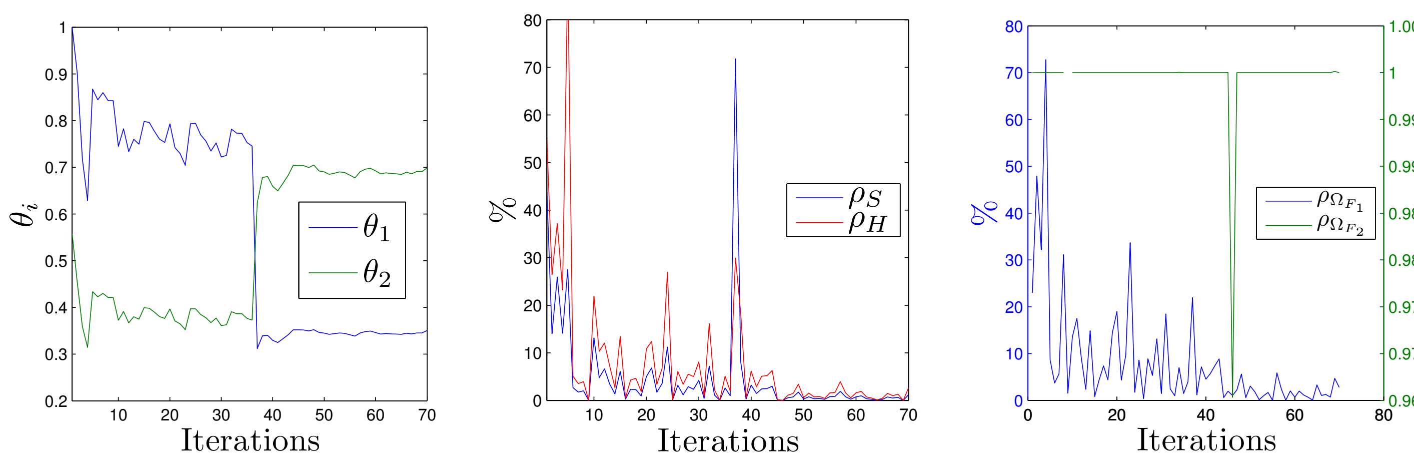


Figure 7: Graphical convergence representation of the proposed methodology for the short column example (left to right: design variables, overall convergence, surrogates convergence).

Table 4: Random variables for the crash-worthiness problem

Variable	Distribution	Mean	Std
$x_i,\ i = [1, \ldots, 7]$	Normal	θ_i	0.03
x_8	Normal	0.345	0.006
x_9	Normal	0.345	0.006
x_{10}	Normal	0	10
x_{11}	Normal	0	10

The problem is defined as follow:

$$\min_{\boldsymbol{\theta}} \quad F(\boldsymbol{\theta})$$

$$\text{s.t.} \quad \mathbb{P}[\mathbf{X} \in \Omega_{F_i}] \leq 0.1 \qquad i = 1, \ldots, 10$$

$$0.5 \leq \theta_i \leq 1.5 \qquad i = 1, \ldots, 7$$

where F is the weight. Ten failure domains are involved: two structural velocities, three deflection along with a viscous criterion constraints on the dummy chest and two forces on the dummy body (Gu, Yang, Tho, Makowskit, Faruquet, & Y. Li 2001). The problem features 11 random variables (Table 4). The means θ_i of the first seven variables are used as optimization variables. An initial CVT DOE of 60 points is used. Each iteration add one samples to each Ω_{F_i} (i.e., 10 samples total). Note that for this problem, the objective function is not approximated by Kriging. The convergence of the algorithm over 200 iterations is depicted in Figure 8 and the results are summarized in Table 5.

At iteration 100, beyond which no further improvement is observed, a total of 1060 (60+ 10 × 100) function calls are required. In addition, Figure 9 shows the evolution of the actual probabilities of failure. Most of the failure domains converge before fifty iterations. Only four failure domains require further refinement. In this work, the fact that some failure domains converged early have not been used to reduce the number of function calls.

Table 5: Results for the crash-worthiness problem at iteration 100

	Optimum	Actual*	Error (%)
θ_1	0.5	0.5	0
θ_2	1.31	1.31	0.12
θ_3	0.5	0.5	0
θ_4	1.31	1.32	0.22
θ_5	0.67	0.68	1.17
θ_6	1.5	1.5	0
θ_7	0.5	0.5	0
F	24.51	24.53	1.91
$P_f^{\star\star}$	0.105	0.1	5
$\beta^{\star\star\star}$	1.25	1.28	2.4

* Obtained through a brute nested RBDO technique
** $P_f = \max\{\mathbb{P}[\mathbf{X} \in \Omega_{F_i}]\}$,*** $\beta = -\Phi\left(\max\{\mathbb{P}[\mathbf{X} \in \Omega_{F_i}]\}\right)$

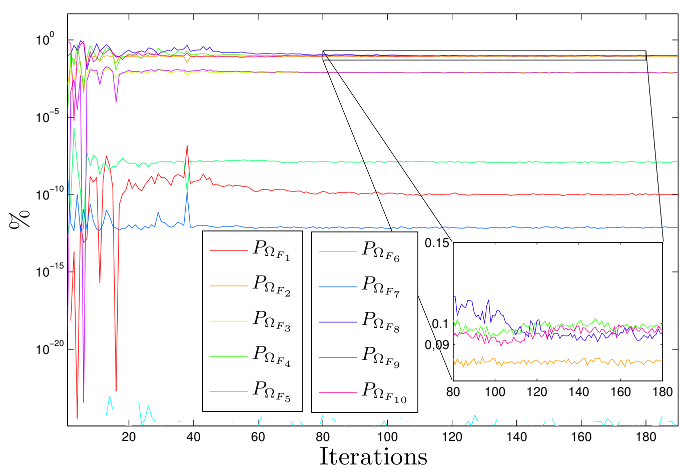


Figure 9: Evolution of the actual probabilities of failure during the RBDO process for the crash-worthiness example.

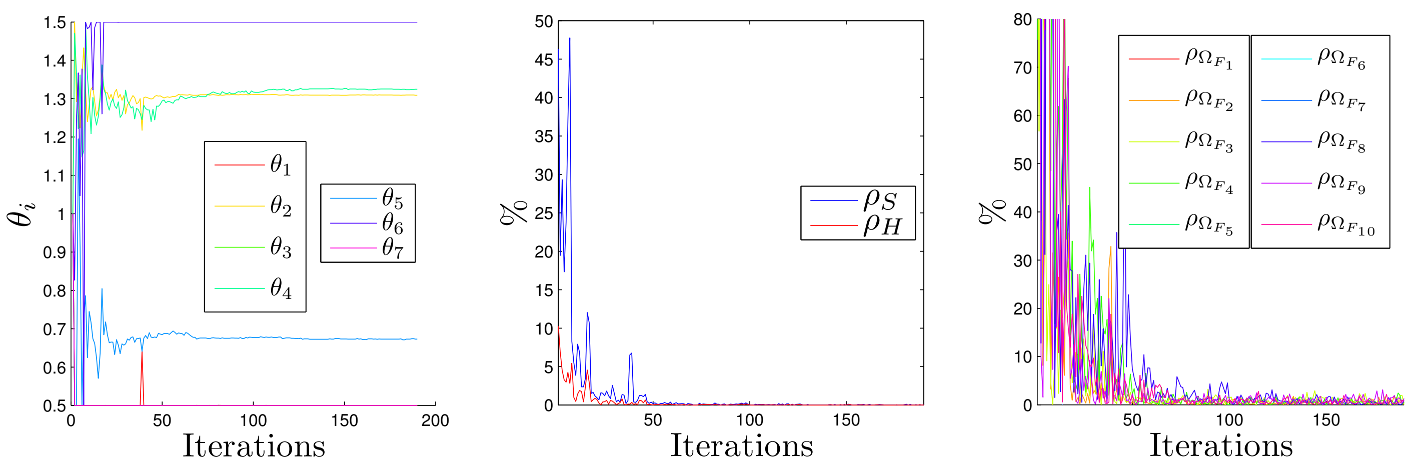


Figure 8: Graphical convergence representation of the proposed methodology for the crash-worthiness example (left to right: design variables, overall convergence, surrogates convergence).

4 CONCLUSION

In this work, a new RBDO approach has been introduced. The approaches hinges on the local refinement of a support vector machine classifier to approximate the failure domain. This refinement is performed through an adaptive sampling scheme that locate samples based on the probabilistic and spatial distributions of the variables and samples. In addition, the probability of failure and its sensitivity is estimated using Subset Simulation. Three test cases were used to demonstrate the efficiency of the proposed scheme. In particular, a problem with 11 dimensions and 10 probabilistic constraints was solved.

Future studies involve the testing of the adaptive sampling scheme with different joint distributions. In addition, the authors will investigate modification of the algorithm (e.g., active set strategy) to further reduce the computational burden. More importantly, a variation will be studied to enable the use of deterministic variables in the optimization problem.

5 ACKNOWLEDGMENTS

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